7. POWER SYSTEM STABILITY

7.1 INTRODUCTION

A large power system consists of a number of synchronous machines (or equipments or components) operating in synchronism. During normal operation i.e, during steady state conditions the different components of power system remain in equilibrium (i.e, synchronism) with respect to each other. When the system is subjected to some form of disturbance, there is a tendency for the system to develop forces to bring it to a normal or stable condition. The ability of a system to reach a normal or stable condition after being disturbed is called stability.

The term stability refers to stable operation of the synchronous machines connected to a power system when they are subjected to sudden disturbances. Hence we can say that the stability is the ability of a power system to return to stable operation when it is subjected to a disturbance. The major disturbances which cause stability problem are the loss of generations, excitations, loss of transmission facilities, switching operations, momentary changes in loads and faults etc.

The basic purpose for conducting stability studies is to see whether the proposed or existing system will remain in stable or equilibrium during or after the disturbance. The data obtained from the stability studies are usually voltages, internal machine angle (i.e, load or torque angle), currents, powers, speeds and torques of the machines as well as voltages at the buses and power flows in the lines of the power system network during and immediately after the disturbances. The data obtained from stability studies help us to design adequate protective scheme of the power system network.

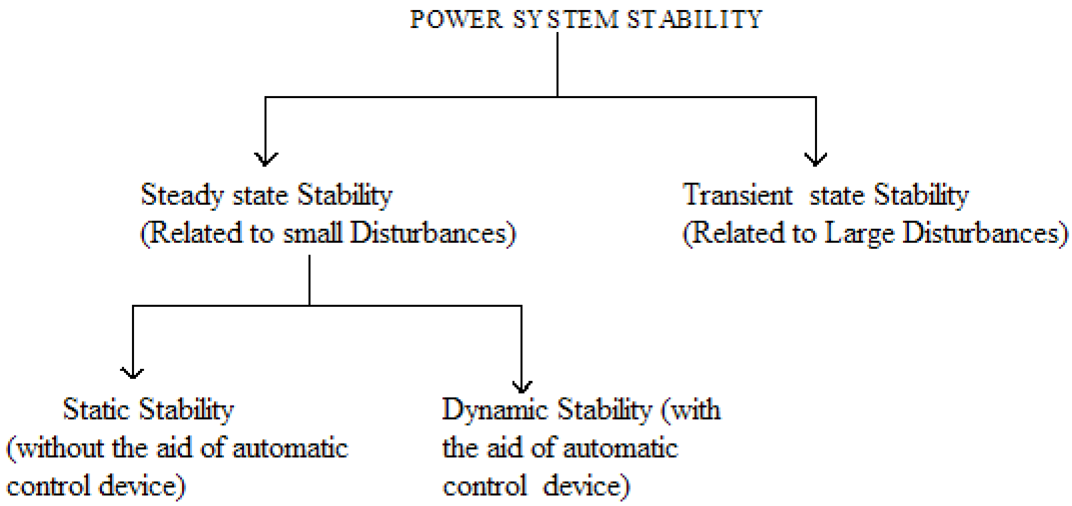
7.2 CLASSIFICATION OF STABILITY STUDIES

When the system is subjected to some form of disturbance, there is a tendency for the system to develop forces to bring it to a normal or stable condition. The ability of a system to reach a normal or stable condition after being disturbed is called stability.

The stability limit is the max power that can be transferred in a network between the sources and loads without loss of synchronism.

Depending on the nature and magnitude of disturbances the stability studies can be classified in to the following types

1. Steady state stability
2. Transient stability



1. Steady state stability

The steady state stability is defined as the ability of a power system to remain stable (i.e., without losing synchronism) for small disturbances (such as gradual changes in load). If the magnitude of disturbance is small such as gradual change in load, the dynamics of rotating machines will not effected and hence dynamical equations of the rotating machines (such as synchronous machines etc) will not appear in the mathematical formulation of the power system for the purpose of stability studies. This is the simplest case of stability and is referred to as steady state stability of the system. Steady state stability is a function of operating conditions only.

The steady state stability limit refers to the maximum power which can be transferred through the system without loss of stability for small disturbances.

Steady state stability is subdivided in to static and dynamic stabilities to make a distinction between operations with and without automatic control devices such as governors and voltage regulators.

Static stability refers to inherent stability that prevails without the aid of automatic control devices.

Dynamic stability refers to artificial stability given to an inherently unstable system by automatic control devices. Dynamic stability is concerned with small disturbances lasting for times of order of 10 to 30 seconds.

2. Transient stability

The transient stability is defined as the ability of a power system to remain stable (i.e., without loosing synchronism) for large disturbances. (Example of large disturbances are sudden change in loads, loss of generations, excitations, transmission facilities, switching operations and faults). Transient stability is a function of both system operating conditions and the disturbances.

The maximum power which can be transferred through the system without the loss of stability under sudden disturbances is referred as transient stability limit.

7.3 POWER ANGLE CURVE

Case (i): Generator loaded at its terminals

The graphical representation of power P_e and the load angle δ is called the power-angle diagram or power angle curve. Such a diagram is widely used in power system stability studies. Fig(7.1) shows a synchronous machine having a direct axis synchronous reactance X_d

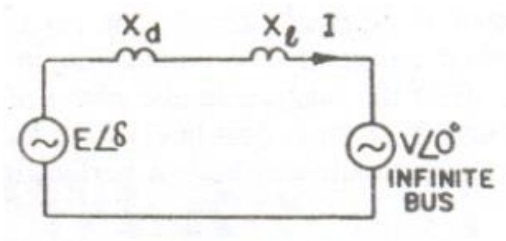


Fig.(7.1): A Synchronous machine loaded at its terminals(here take X_l=0)

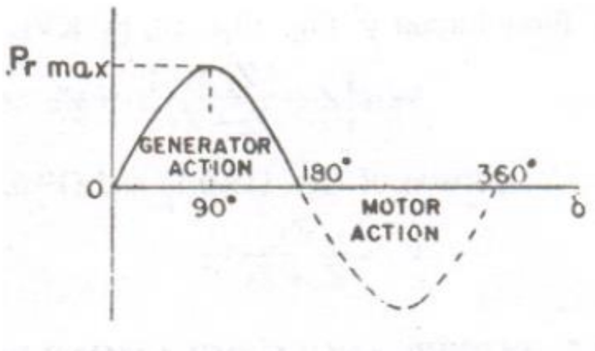


Fig.(7.2): Power angle curve

Let E= $E\angle\delta$ =voltage behind direct axis synchronous reactance of generator

V= $V\angle 0$ terminal voltage of generator.

The complex power output of generator is

$$S=VI^* =P_e+jQ_e \qquad \text{--- (7.1)}$$

$$=|V|\left[\frac{|E|\angle\delta-|V|}{jX_d}\right]^*$$

$$= |V|\left[\frac{|E|\angle\delta}{X_d\angle 90} + j\frac{|V|}{X_d}\right]^*$$

$$=|V|\left[\frac{|E|\angle-\delta}{X_d\angle-90} - j\frac{|V|}{X_d}\right]$$

$$=\frac{|E||V|}{X_d}\angle 90-\delta - j\frac{|V|^2}{X_d}$$

$$P_e + jQ_e = \frac{|E||V|}{X_d}\sin\delta + j\left[\frac{|E||V|}{X_d}\cos\delta - \frac{|V|^2}{X_d}\right] \quad \text{--- (7.2)}$$

The real power output, P_e of the generator is given by

$$P_e = \text{R.P of (S)} = \frac{|E||V|}{X_d}\sin\delta \quad \text{--- (7.3)}$$

Thus, the real power output depends on $|E|, |V|, X_d$ and power angle δ.

The reactive power output , Q_e of the generator is given by

$$Q_e = \text{I.P of (S)} = \frac{|E||V|}{X_d}\cos\delta - \frac{|V|^2}{X_d} \quad \text{--- (7.4)}$$

If line to line values of E and V are used in eqns.(7.3) and(7.4), we directly get 3-Φ power.

Fig.(7.2) shows the steady-state real power variation with power angle for both generator and motor action for constant values of E,V and X_d. This curve is known as power angle curve. The condition of positive value of δ ,i..e E leading V, applies to the generator action and the condition of negative value of δ ,i.e, E lagging V applies to the motor action.

The maximum steady-state power transfer occurs when δ=90

From eqn.(7.3), $P_{e,\max} = \frac{|E||V|}{X_d}\sin 90 = \frac{|E||V|}{X_d}$

Maximum power is transferred when $\delta=90^0$. As δ is increased beyond 90^0 ,P_e decreases and becomes zero at $\delta=180^0$. Beyond $\delta=180^0$,P_e becomes negative which implies that the power flow direction is reversed and the power is supplied from the infinite bus to the generator. The value of $P_{e,max}$ is often called the pull-out or steady-state limit. In actual practice δ is kept round 30^0

Assume that the generator is working under steady state conditions and the power angle δ increases by a small amount Δδ. The increase in synchronous power output is given by

$$\Delta\text{P} = \frac{\partial P}{\partial\delta}(\Delta\delta) = \text{P}_\text{r}(\Delta\delta)$$

$$P_r = \frac{\Delta P}{\Delta \delta} = \frac{\partial P}{\partial \delta} = \frac{|E||V|}{X_d}\cos\delta \quad \text{--- (7.5)}$$

Where P_r synchronizing power coefficient

Case (ii): Generator connected to Infinite bus.

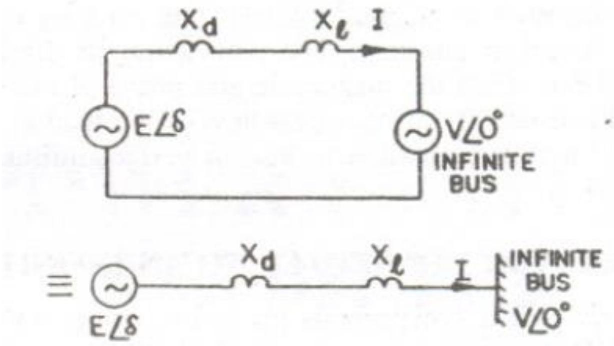


Fig.(7.3): A synchronous machine connected to infinite bus

The above fig.(7.3) shows a synchronous machine connected to an infinite bus through a transmission line of reactance X_l. Let us assume that the line resistance and capacitance are neglected.

Let V=V $\angle 0$ =voltage of infinite bus

E=E $\angle\delta$ =voltage behind direct axis synchronous reactance of the machine.

The complex power o/p the generator is

$$S = V I^* = P_e + jQ_e$$

$$= |V|\left[\frac{|E|\angle\delta - |V|\angle 0}{jX}\right]^*$$

$$= |V|\left[\frac{|E||\delta|}{X\angle 90} + j\frac{|V|\angle 0}{X}\right]^*$$

$$= |V|\left[\frac{|E|}{X}\angle 90 - \delta - j\frac{|V|}{X}\right]$$

$$= \frac{|E||V|}{X}\angle 90 - \delta - j\frac{V^2}{X}$$

$$P_e + jQ_e = \frac{|E||V|}{X}\sin\delta + j\left[\frac{|E||V|}{X}\cos\delta - \frac{V^2}{X}\right] \quad \text{--- (7.6)}$$

The real power output of the generator is

$$P_e = \frac{|E||V|}{X}\sin\delta = P_{\max}\sin\delta \quad \text{--- (7.7)}$$

The maximum steady state power transfer P_{max} occurs when ,δ=90^0 and equals to $\frac{|E||V|}{X}$

$$\therefore P_{max} = \frac{|E||V|}{X} \qquad \text{--- (7.8)}$$

Transfer reactance: The total reactance X between two voltage sources V and E is called the transfer reactance. It is seen that the maximum power limit is inversely proportional to the transfer reactance.

Case (iii) :Power transfer through Impedance

In all electrical machines and transmission lines, the resistance is negligible as compared to inductive reactance. However, it is instructive to study the power transfer through an impedance Z.

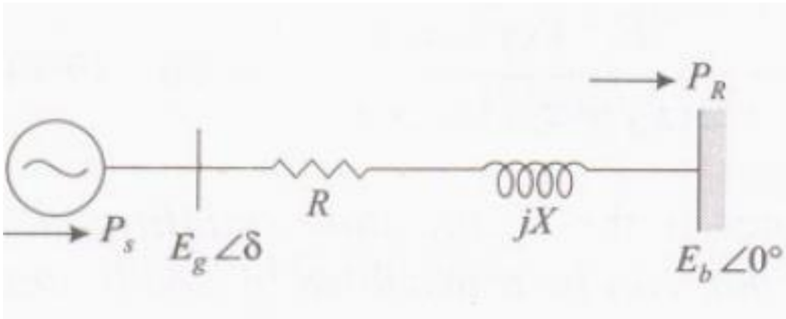


Fig.(7.4)

The complex power received by an infinite bus is

$$S_2 = P_2 + jQ_2$$

$$= V_2 I^*$$

$$= VI^*$$

$$= V\left[\frac{E\angle\delta - V\angle 0}{Z\angle\theta}\right]^*$$

$$= V\left[\frac{E}{Z}\angle\delta-\theta - \frac{V}{Z}\angle -\theta\right]^*$$

$$= V\left[\frac{E}{Z}\angle\theta-\delta - \frac{V}{Z}\angle\theta\right]$$

$$P_e + jQ_e = \frac{EV}{Z}\angle\theta-\delta - \frac{V^2}{Z}\angle\theta \qquad \text{--- (7.9)}$$

Active power received by infinite bus

$$P_e = P_2 = \frac{EV}{Z}\cos(\theta-\delta) - \frac{V^2}{Z}\cos\theta$$

$$= \frac{EV}{Z}\cos(90-\alpha-\delta) - \frac{V^2}{Z}\cos(90-\alpha)$$

$$\Rightarrow P_e = \frac{EV}{Z}\sin(\delta+\alpha) - \frac{V^2}{Z}\sin\alpha \qquad \text{--- (7.10)}$$

The angle, α is a function of the impedance of the line therefore, the power received P_e, is maximum, when $\alpha + \delta = 90^0$

$$P_{e,\max} = \frac{EV}{Z} - \frac{V^2}{Z}\sin\alpha$$

From impedance triangle

$$\sin\alpha = R/Z, \cos\alpha = X/Z$$

$$\therefore P_{e,\max} = \frac{EV}{\sqrt{R^2+X^2}} - \frac{V^2 R}{\sqrt{R^2+X^2}\sqrt{R^2+X^2}}$$

If E=V, then

$$P_{e,\max} = \frac{V^2}{\sqrt{R^2+X^2}} - \frac{V^2 R}{R^2+X^2}$$

For $P_{e,max}$ to be maximum (i.e. for max. power transfer)

$$\frac{dP_{e,\max}}{dX} = 0 \Rightarrow \frac{d}{dX}\left[\frac{1}{\sqrt{R^2+X^2}} - \frac{R}{R^2+X^2}\right] = 0$$

$$\Rightarrow -\frac{1\times 2X}{2(R^2+X^2)^{3/2}} + \frac{R\times 2X}{(R^2+X^2)^2} = 0$$

$$\Rightarrow \frac{1}{2(R^2+X^2)\sqrt{R^2+X^2}} = \frac{R}{(R^2+X^2)^2}$$

$$\Rightarrow \sqrt{R^2+X^2} = 2R$$

$$\Rightarrow R^2+X^2 = 4R^2$$

$$\Rightarrow 3R^2 = X^2$$

$$\Rightarrow X = \sqrt{3}R \qquad \text{--- (7.11)}$$

In actual practice R is very small as compared to X and therefore, the practical application of equation is limited. Eqn.(7.11) shows that if X=0, power transferred is zero. Thus a finite value of reactance is necessary for power transfer.

7.4 STABILITY LIMITS

The stability limit is the max. power that can be transferred in a network between sources and loads without loss of synchronism.

The steady state stability limit is the max. power that can be transferred without the system becoming unstable, when the load is increased gradually, under steady state condition.

Transient stability limit is the max. power that can be transferred with out the system becoming unstable when a sudden or large disturbance occurs.

The system experiences a shock by sudden and large power changes and violent fluctuations of voltage will occur. Consequently, individual machines or group of machines may go out of step. The rapidity of the application of a large disturbance is responsible for loss of stability otherwise it may be possible to maintain stability if the same large load is applied gradually. Thus the transient stability is lower than the steady-state stability.

7.5 POWER ANGLE EQUATION (OR STEADY STATE STABILITY LIMIT) IN TERMS OF ABCD PARAMETERS

Consider a simple system consisting of a synchronous generator connected to an infinite bus through a network represented by the ABCD Parameters as shown in fig.(7.5)

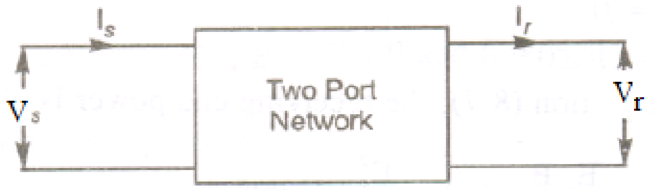


Fig.(7.5) : Synchronous generator connected to an infinite bus through a two port network

The sending end and the receiving end voltages are assumed as

$$V_s = E\angle\delta, V_r = V\angle 0$$

We have $V_s = AV_r + BI_r$

$$\Rightarrow I_r = \frac{V_s}{B} - \frac{A}{B}V_r$$

$$= \frac{E\angle\delta}{B\angle\beta} - \frac{A\angle\alpha}{B\angle\beta}V\angle 0$$

$$I_r = \frac{E}{B}\angle\delta - \beta - \frac{AV}{B}\angle\alpha - \beta$$

The complex power received at infinite bus

$$S_r = V_r I_r^* = P_r + jQ_r$$

$$= V\angle 0\left[\frac{E}{B}\angle\delta - \beta - \frac{AV}{B}\angle\alpha - \beta\right]^*$$

$$\Rightarrow P_r + jQ_r = V\angle 0\left[\frac{E}{B}\angle\beta - \delta - \frac{AV}{B}\angle\beta - \alpha\right]$$

$$= \frac{EV}{B}\angle\beta - \delta - \frac{AV^2}{B}\angle\beta - \alpha$$

$$= \frac{EV}{B}\left[\cos(\beta-\delta)+j\sin(\beta-\alpha)\right]-\frac{AV^2}{B}\left[\cos(\beta-\alpha)+j\sin(\beta-\alpha)\right]$$

By equating real and imaginary parts on both sides

$$P_r = \frac{EV}{B}\cos(\beta-\delta)-\frac{AV^2}{B}\cos(\beta-\alpha)$$

$$Q_r = \frac{EV}{B}\sin(\beta-\delta)-\frac{AV^2}{B}\sin(\beta-\alpha)$$

The power received is maximum when β=δ

$$\therefore P_{\max} = \frac{EV}{B}-\frac{AV^2}{B}\sin(\beta-\alpha)$$

7.6 DYNAMICS OF SYNCHRONOUS MACHINE

The kinetic energy of the rotor of the synchronous machine is

$$KE = \frac{1}{2}J\omega_{sm}^2 \times 10^{-6}\, MJ$$

$$where\ J = rotor\ moment\ of\ inertia\ in\ kg-m^2$$

$$\omega_{sm} = synchronous\ speeed\ in\ rad(elec)/\sec$$

But $\omega_s = \left(\frac{P}{2}\right)\omega_{sm} = rotor\ speed\ in\ rad(elec)/\sec$

where P = no. of poles in machine

$$KE = \left[\frac{1}{2}J\left(\frac{2}{P}\right)^2\omega_s^2\right]\times 10^{-6}$$

$$= \frac{1}{2}\left[J\times\left(\frac{2}{P}\right)^2\omega_s\times 10^{-6}\right]\times\omega_s$$

$$KE = \frac{1}{2}M\omega_s \qquad \text{--- (7.12)}$$

Where $M = J\times\left(\frac{2}{P}\right)^2\omega_s\times 10^{-6}$ = Moment of inertia in MJ.sec/elec.rad

We shall define the inertia constant H such that

$$GH = KE = \frac{1}{2}M\omega_s \quad MJ \qquad \text{--- (7.13)}$$

Where G = machine rating in MVA

H = inertia constant in MJ/MVA

$$GH = \frac{1}{2} M \omega_s$$

$$\Rightarrow M = \frac{2GH}{\omega_s} = \frac{GH}{\prod f} MJ.\sec/ elec.rad = \frac{GH}{180 f} MJ.\sec/ elec.\deg ree$$

Where M=the inertia constant

Taking G as base the inertia constant in p.u is

$$M_{(p.u)} = \frac{H}{\prod f}$$

$$= \frac{H}{180 f} \quad \text{--- (7.14)}$$

The inertia constant H has a characteristic value or a range of values for each class of machines.

7.7 SWING EQUATION

The behaviour of a synchronous machine during transients is described as swing equation. Fig.(7.6) shows the torque, speed and flow of a mechanical and electrical powers in a synchronous a machine. It is assumed that the windage, friction and iron loss is negligible.

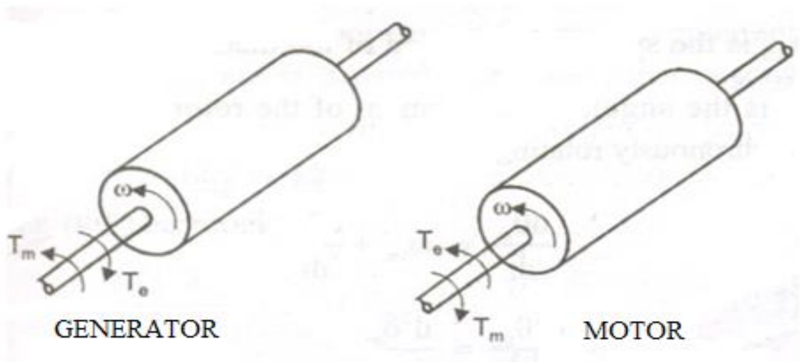


Fig.(7.6)

Under steady state operating condition the T_e and T_m are equal and machine runs at constant speed. If there is a difference between the two torques then the rotor will have an accelerating or decelerating torque

$T_a = T_m - T_e$ --- (7.15)

By Newton's second law of motion we can say that the accelerating torque, T_a is directly proportional to angular acceleration.

$$\therefore T_a \, \alpha \, \frac{d^2\theta_m}{dt^2} \Rightarrow \quad T_a = J \frac{d^2\theta_m}{dt^2}$$

$$J \frac{d^2\theta_m}{dt^2} = (T_m - T_e) = T_a \; N - m \quad \text{--- (7.16)}$$

Where J = The total moment of inertia of the rotor mass in kg - m^2

θ_m= Angular displacement of the rotor with respect to a stationary axis in mechanical radians.

t=Time, in seconds.

T_m= The mechanical (or) shaft torques supplied by the prime mover in N-m.

T_e= Net electrical (or) electromagnetic torque in N - m.

T_a= Net accelerating torque in N - m.

Multiplying eqn.(7.1) by ω_{sm}

$$J\omega_{sm}\frac{d^2\theta_m}{dt^2}\times 10^{-6} = \left(P_m - P_e\right) \qquad MW \qquad \text{--- (7.17)}$$

Where P_m= Mechanical power input in MW

P_e= Electrical power output in MW; stator copper loss is assumed to be negligible

But $\omega_{sm} = \left(\frac{2}{P}\right)\omega_s$

$$\theta_m = \frac{2}{P}\theta_e \Rightarrow \frac{d\theta_m}{dt} = \frac{2}{P}\frac{d\theta_e}{dt} \Rightarrow \frac{d^2\theta_m}{dt^2} = \frac{2}{P}\frac{d^2\theta_e}{dt^2} \qquad \text{--- (7.18)}$$

From eqns. (7.17) and (7.18)

$$J\times\left(\frac{2}{P}\right)\omega_s\times\left(\frac{2}{P}\right)\frac{d^2\theta_m}{dt^2}\times 10^{-6} = (P_m - P_e)MW$$

$$\left[J\left(\frac{2}{P}\right)^2\omega_s\times 10^{-6}\right]\frac{d^2\theta_e}{dt^2} = (P_m - P_e)MW$$

$$\Rightarrow M\frac{d^2\theta_e}{dt^2} = P_m - P_e \qquad \text{--- (7.19)}$$

Where θ_e=angle in rad(elect)

It is more convenient to measure the angular position of the rotor w.r.t a synchronously rotating frame of reference. The angular displacement θ_e and δ_m are related to synchronous speed by the following equation

$$\delta=\theta_e - \omega_s t \qquad \text{--- (7.20)}$$

Differentiating both sides w.r.t 't'

$$\frac{d\delta}{dt} = \frac{d\theta_e}{dt} \Rightarrow \frac{d^2\delta}{dt^2} = \frac{d^2\theta_e}{dt^2} \qquad \text{--- (7.21)}$$

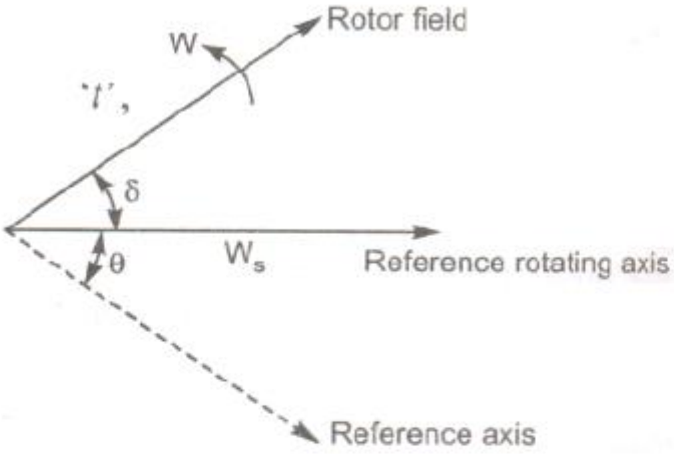


Fig.(7.7)

From eqns.(7.19) and (7.21)

$$M\frac{d^2\delta}{dt^2} = P_m - P_e$$

$$\frac{GH}{\Pi f}\frac{d^2\delta}{dt^2} = (P_m - P_e)MW \qquad \text{--- (7.22)}$$

Dividing throughout by G, the MVA rating of the machine, we can get

$$\frac{H}{\Pi f}\frac{d^2\delta}{dt^2} = (P_m - P_e)\ p.u$$

$$\Rightarrow M_{(p.u)}\frac{d^2\delta}{dt^2} = P_m - P_e \qquad \text{--- (7.23)}$$

The above equation is called swing equation of the machine, is the fundamental equation which governs the rotational dynamics of the synchronous machine in stability studies. It is a second order differential equation where the damping term (proportional to dδ/dt) is absent because of the assumption of a lossless machine and the damper winding has been ignored.

When swing equation is solved we obtain the expression for δ as a function of time. A graph of solution is called the swing curve of the machine and the inspection of the swing curves of all the machines of the system will show whether the machines remain in synchronism or not after a disturbance.

Multi machine system

In a multi machine system a common system base must be chosen

Let G_{mach}=machine MVA rating (base)

G_{system}=system MVA base

Multiplying the eqn.(7.23) by $\frac{G_{mach}}{G_{sys}}$ on both sides, we get

$$\frac{G_{mach}}{Gs_{ys}}\left(\frac{H_{mach}}{\pi f}\cdot\frac{d^2\delta}{dt^2}\right)=\left(P_m-P_e\right)\frac{G_{mach}}{G_{sys}}$$

$$\frac{H_{sys}}{\pi f}\frac{d^2\delta}{dt^2}=(Pm-Pe)\,pu(in\,system\,base) \qquad \text{--- (7.24)}$$

$$where\;\; H_{sys}=H_{mach}\left(\frac{G_{mach}}{G_{sys}}\right)$$

= Machine inertia constant in system base

Machines swinging coherently

The swing equations of two machines on a common system base

$$\frac{H_1}{\pi.f}\frac{d^2\delta_1}{dt^2}=(P_{m1}-P_{e1})\,pu \qquad \text{--- (7.25)}$$

$$\frac{H_2}{\pi.f}\frac{d^2\delta_2}{dt^2}=(P_{m2}-P_{e2})\,pu \qquad \text{--- (7.26)}$$

Since the machine rotors swing together (coherently)

$$\delta_1=\delta_2=\delta$$

By adding eqns.(7.25) & (7.26), we get

$$\left(\frac{H_1}{\pi.f}+\frac{H_2}{\pi.f}\right)\frac{d^2\delta}{dt^2}=(P_{m1}+P_{m2})-(P_{e1}-P_{e2})$$

$$\Rightarrow\frac{H_{eq}}{\pi.f}\frac{d^2\delta}{dt^2}=P_m-P_e \qquad \text{--- (7.27)}$$

Where $P_m=P_{m1}+P_{m2}$

$P_e=P_{e1}+P_{e2}$

$H_{eq}=H_1+H_2$ --- (7.28)

The two machines swing coherently are thus reduced to a single machine as in eqn.(7.27). The equivalent inertia in eqn.(7.28) can be written as

$$H_{eq}=\frac{H_{1mach}G_{1mach}}{G_{sys}}+\frac{H_{2mach}G_{2mach}}{G_{sys}}$$

The above results are easily extendable to any no. of machines swinging coherently.

Problem-1: A 2 pole, 50 Hz, 60 MVA turbo generator has a moment of inertia of 9×10^3 kg-m^2. Calculate a) the kinetic energy in MJ at rated speed. b) the inertia constant M and H c)the inertia constant on 50MVA base

Solution: Given that

P=2, f=50 Hz,

$$J = 9 \times 10^3 kg - m^2$$
$$G = 60MVA$$
$$N = \frac{120f}{P} = 120 \times \frac{50}{2} = 3000rpm$$
$$\omega_{sm} = \frac{2.\pi.N}{60} = \frac{2.\pi \times 3000}{60} = 314$$

a) KE stored = $\frac{1}{2}J\omega_{sm}^2 = \frac{1}{2}9 \times 10^3 \times (314)2 = 444MJ$

b) We know that,

KE=GH=$\frac{1}{2}M\omega_s$

H=$\frac{KE}{G} = \frac{444}{60} = 7.4MJ/MVA$

$$\omega_s = \frac{P}{2} \times \omega_{sm} = \frac{P}{2} \times \frac{2.\pi.N}{60} = \frac{P}{2} \times \frac{2.\pi}{60} \times \frac{120f}{P} = 2.\pi.f$$

We also know that,

KE=$\frac{1}{2}M\omega_s \Rightarrow M = \frac{2KE}{\omega_s} = \frac{2KE}{2.\pi.f} = \frac{444}{3.14 \times 50} = 2.828MJ\,\text{sec}/elec - rad$

c) Per unit inertia constant,

$$M_{pu} = \frac{M}{base\,MVA} = \frac{2.828}{50} = 0.0565pu$$

Problem-2: A 4 pole, 50 Hz, turbo generator rated at 100MVA, 11kV has an inertia constant of 8 MJ/MVA

a) Find the stored energy in the rotor at synchronous speed

b) If the mechanical input is suddenly raised to 80MW for an electrical load of 50MW, find rotor acceleration, neglecting mechanical and electrical losses.

c) If the acceleration calculated in part(b) is maintained for 10 cycles, find the change in torque angle, rotor speed in rpm at the end of this period.

Solution: Given that

f=50Hz, P=4, G=100MVA

V=11KV, H=8MJ/MVA

a) Stored energy KE=GH=100 x 8=800MJ

b) $P_a = P_m - P_e = 80 - 50 = 30$ MW

$$M \times \frac{\partial^2 \delta}{\partial t^2} = 30MW$$

$$M = \frac{GH}{\prod f} = \frac{800}{180 \times 50} = \frac{4}{45} MJ \sec/ elec - \deg$$

$$= \frac{800}{\pi \times 50} = \frac{16}{\pi} MJ \sec/ elec - rad$$

$$\frac{4}{45} \times \frac{d^2\delta}{dt^2} = 30 \Rightarrow \frac{\partial^2 \delta}{\partial t^2} = \alpha = \frac{30 \times 45}{4} = 337.5 elec - \deg ree / \sec^2$$

c) Change in torque angle and rotor speed

$$\frac{\partial^2 \delta}{\partial t^2} = \frac{P_a}{M} = \frac{30 \times \pi}{16} = 5.89$$

Taking integration on both sides on both sides,

$$\frac{d\delta}{dt} = 5.89t$$

Taking integration once again on both sides on both sides

$$\delta = 2.945t^2$$

We know that frequency f=50Hz=50cycles/sec

i.e. 50 cycles--------1sec

10 cycles--------10/50=0.2sec = t

$$\therefore \delta = 2.945t^2 = 2.945 \times 0.2^2 = 0.1178 elec - rad$$

$$= 0.1178 \times \frac{180}{\pi} = 6.75 elec - \deg ree$$

$$\frac{d\delta}{dt} = 5.89t = 5.89 \times 0.2 = 1.177 rad / \sec$$

$$= \frac{1.177}{2\prod} \times 60 rpm = 11.2 rpm$$

$\therefore$ Rotor speed at the end of 10 cycles

$$= \frac{120 f}{P} + 5.619 = \frac{120 \times 50}{4} + 11.2 = 1511.219 rpm$$

Problem-3: A 50 Hz synchronous generator is connected to an infinite bus through a line. The p.u reactance of generator and line are j0.3pu and j0.2pu respectively. The generator no load voltage is 1.1pu and that of infinite bus is 1.0pu. The inertia constant of generator is 3MJ/MVA. Determine the frequency of natural oscillation if the generator is loaded to (i) 60% (ii) 75% of its max power transfer capacity.

Solution: Given data

Total reactance X=0.3+0.2=0.5

E=1.1pu, V=1.0pu, H=3MJ/M

(i) For 60% loading

$$P_e = P_{\max} \sin\delta \Rightarrow \sin\delta = \frac{P_e}{P_{\max}} = \frac{0.6P_{\max}}{P_{\max}} = 0.6 \Rightarrow \delta = 36.86^0$$

$$f_n = \left[\left(\frac{\partial P_e}{\partial \delta}\right)_0 / M\right]^{1/2}$$

$$\left(\frac{\partial P_e}{\partial \delta}\right)_0 = \frac{EV}{X}\cos\delta = \frac{1.1\times 1}{0.5}\times 0.8 = 1.76$$

$$M = \frac{GH}{\pi . f} = \frac{1\times 3}{50.\pi} = \frac{3}{50\pi}$$

$$f_n = \left(\frac{1.76\times 3}{50\pi}\right)^{1/2} = 9.6\,rad/\sec \; = \frac{9.6}{2\pi} = 1.53Hz$$

(ii) For 75% loading

$$\sin\delta = \frac{P_e}{P_{\max}} = 0.75 \Rightarrow \delta = 48.59^0$$

$$\left(\frac{\partial P_e}{\partial \delta}\right)_0 = \frac{EV}{X}\cos\delta = \frac{1.1\times 1}{0.5}\times 0.6614 = 1.455$$

$$f_n = \left(\left(\frac{\partial P_e}{\partial \delta}\right)_0 \Big/ M\right)^{1/2} = \left(\frac{1.455\times 3}{50\pi}\right)^{1/2} = 8.726\,rad/\sec \quad = \frac{8.726}{2\pi} = 1.39\,Hz$$

7.8 SWING CURVES

The swing equation (7.23) is a non-linear differential equation of the second order. The solution of which gives the relationship between torque or load angle (δ) usually in radians and time in seconds. The graph between load angle and time is called Swing Curve. Swing curves provide information regarding stability. The fig.(7.8) show the tendency of δ to oscillate or increase beyond the point of return. If δ increases continuously with time the system is unstable. While if δ starts decreasing after reaching a maximum value it is said that the system will remain stable.

Swing curves are useful in determining the adequacy of relay protection on power systems with regard to the clearing of faults before one or more machines become unstable and fall out of synchronism, The critical clearing time is found to specify the correct speed of the circuit breaker. The solution of swing equation involves elliptic integrals. Step-by-step

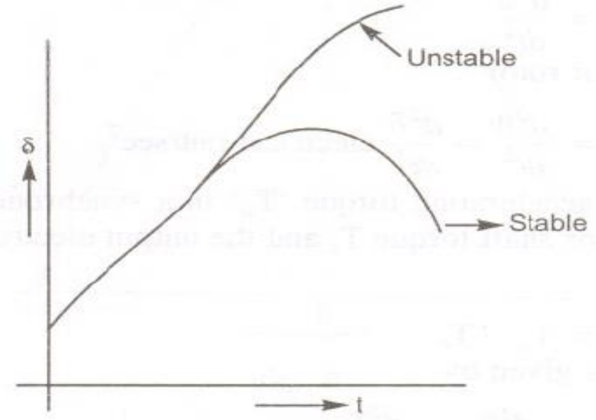


Fig.(7.8): Swing curve

method may be used for numerical solution of swing equation. Nowadays digital computers are being employed to solve swing equation for a multi-machine system. However for a two machine system, there is a graphical method of determining whether the two machines are running at stand-still with respect to each other. The method can also be used to access the transient stability of a single machine system connected to infinite bus bars. This method is known as equal area criterion for stability, which eliminates the actual solving of swing equation.

7.9 ANALYSIS OF STEADY STATE STABILITY

The steady state stability limit of a particular circuit of a power system is defined as the max. power that can be transmitted to receiving end with out loss of synchronism.

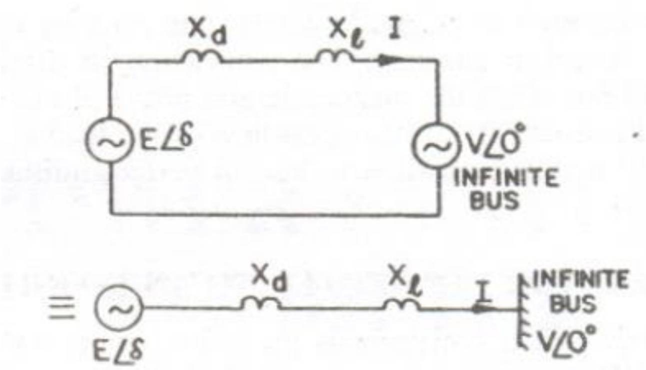


Fig.(7.9): Synchronous machine connected to an infinite bus

Consider the simple system shown in fig.(7.9), whose dynamics is described by equations.

$$M\frac{d^2\delta}{dt^2} = (P_m - P_e) \qquad \text{--- (7.29)}$$

Where $M = \dfrac{H}{\Pi f}$ in pu system

And $P_e = \frac{|E||V|}{X} \sin\delta = P_{\max} \sin\delta$ --- (7.30)

Let the system be operating with steady power transfer with a torque angle δ_0. In this operating condition power output P_{e0}. Now the mechanical power input P_m is equal to P_{e0} under ideal condition.

i.e. $P_m=P_{e0}=P_{max} \sin\delta_0$ --- (7.31)

With power input, P_m remaining same let us assume that the electrical power output increases by a small amount ΔP. Now the torque angle is changed by a small amount $\Delta\delta$. Therefore the new value of torque angle is $(\delta_0+\Delta\delta)$

The electrical power output for this new torque angle $(\delta_0+\Delta\delta)$ is given by

$$P_e^1 = P_{e0} + \Delta P = P_{\max} \sin(\delta_0 + \Delta\delta) \quad [\because \textit{from eqn.}(7.30)]$$

$$P_{e0} + \Delta P = P_{\max}\left[\sin\delta_0 \cos\Delta\delta + \cos\delta_0 \sin\Delta\delta\right] \qquad \text{--- (7.32)}$$

Since $\Delta\delta$ is small Sin $\Delta\delta \approx \Delta\delta$

Cos $\Delta\delta \approx 1$

$$\therefore P_{e0} + \Delta P = P_{\max} \sin\delta_0 + P_{\max} \Delta\delta \cos\delta_0 \qquad \text{--- (7.33)}$$

From eqns.(7.32) & (7.33)

$P_{e0}+\Delta P=P_{e0}+(P_{max} \cos \delta_0)\Delta\delta$

$\Delta P=(P_{max} \cos \delta_0)\, \Delta\delta$ --- (7.34)

$$\Rightarrow \frac{\Delta P}{\Delta\delta} = P_{\max} \cos\delta_0 = \left(\frac{\partial Pe}{\partial\delta}\right)_0 \qquad \text{--- (7.35)}$$

From the swing equation,

$$M \frac{d^2\delta}{dt^2} = P_m - P_e$$

For a torque angle of $\delta=\delta_0+\Delta\delta$, the electrical power P_e in the eqn.(7.29) should be replaced by $(P_{c0}+\Delta P)$

$$\therefore M \frac{d^2}{dt}\left(\delta_0 + \Delta\delta\right) = P_m - (P_{e0} + \Delta P)$$

Since δ_0 is constant and $P_m=P_{e0}$, the above equation can be written as

$$M \frac{d^2\Delta\delta}{dt^2} = -\Delta P \qquad \text{--- (7.36)}$$

From eqns. (7.34) & (7.36)

$$M \frac{d^2\Delta\delta}{dt^2} = -(P_{\max} \cos\delta_0)\Delta\delta$$

$$M\frac{d^2}{dt^2}\Delta\delta + P_{\max}\cos\delta_0\Delta\delta = 0 \qquad \text{--- (7.37)}$$

Let $M\frac{d^2}{dt^2} = x^2,\ P_{\max}\cos\delta_0 = C$

$$\therefore Mx^2\Delta\delta + C\Delta\delta = 0$$
$$(Mx^2 + C)\Delta\delta = 0 \qquad \text{--- (7.38)}$$

In the above equation since $\Delta\delta \neq 0$

$$\therefore Mx^2 + C = 0$$

$$x^2 = \frac{-C}{M} \Rightarrow x = \pm\sqrt{-\frac{C}{M}} = \pm\sqrt{\frac{-P_{\max}\cos\delta_0}{M}}$$

$$\Rightarrow x = \pm\sqrt{\frac{-P_{\max}\cos\delta_0}{M}} = \pm\sqrt{\frac{-\left(\frac{\partial P_e}{\partial\delta}\right)_0}{M}} \qquad \text{--- (7.39)}$$

Case (i) : When C is +ve(i.e. $\left(\frac{\partial P_e}{\partial\delta}\right)_0 > 0$ or $P_{\max}\cos\delta_0 > 0$)

In this case the roots are purely imaginary and conjugate. Hence the system behaviour is oscillatory about δ_0. In this analysis the resistances in system have been neglected. If we include the resistances in analysis then the roots will be complex conjugate and the response will be damped oscillatory. Therefore in a practical system, the system is stable for small increment in power provided.

$$P_{\max}\cos\delta_0 > 0 \text{ or } \left(\frac{\partial P_e}{\partial\delta}\right)_0 > 0$$

Case (ii) : When C is –ve (i.e $\left(\frac{\partial P_e}{\partial\delta}\right)_0 < 0$ or $P_{\max}\cos\delta_0 < 0$)

In this case the roots are real and equal in magnitude. One of the root is +ve and other is –ve. Due to the +ve root the torque angle increases with out bound. When there is a small increment in power and machine will loose synchronism. Hence the machine becomes unstable for small changes in power provided.

$$P_{\max}\cos\delta_0 < 0 \text{ or } \left(\frac{\partial P_e}{\partial\delta}\right)_0 < 0$$

Here $\left(\frac{\partial P_e}{\partial\delta}\right)_0$ is known as **synchronizing coefficient**. This is also called **stiffness of Synchronous machine.**

Assuming |E| and |V| to remain constant, the system is unstable if

$$\frac{|E||V|}{X}\cos\delta_0 < 0 \qquad \text{(Or)} \quad \delta_0 > 90$$

The max .power that can be transmitted without loss of stability occurs for

$$\delta_0 = 90$$

and is given by

$$P_{\max} = \frac{|E||V|}{X}$$

Assumptions:

- Generators are represented by constant impedances in series with no load voltages.
- The mechanical power input is constant.
- Damping is negligible.
- Load angle variations are small.
- Speed variations are negligible.

7.10 TRANSIENT STABILITY

Transient stability limit is the maximum power that can be transferred between the sources and sinks without the system becoming unstable when a sudden or large disturbance occurs.

Assumptions:

1. Transmissions line as well as synchronous machine resistance is neglected.
2. Damping term contributed by synchronous machine damper winding is neglected.
3. Rotor speed is assumed to be synchronous.
4. Mechanical power input to machine remains constant.
5. Voltage behind transient reactance is assumed remains constant.
6. Loads are modelled as constant admittances.

The transient stability can be analysed by following methods

i) Equal Area criterion.

ii) Point by point method

iii) Runga-Kutta method

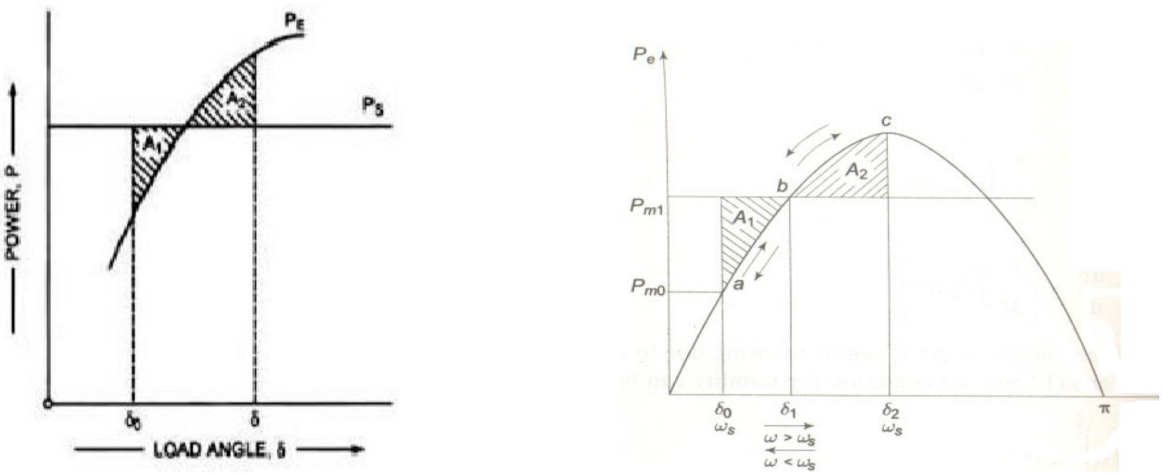
7.11 EQUAL AREA CRITERION

The stability of a single machine connected to an infinite bus can be studied by the use of equal area criterion. Fig (7.10) shows the power-angle curve for an equivalent generator (representing a power export area) connected to an infinite bus.

Suppose that the system is operating under steady-state conditions at a power-angle δ_0 when a large local load ΔP within the power exporting area is switched off. Assume that the mechanical input remains the same, the excess of input over output is ΔP initially. The rotor is accelerated ($P_a = P_m - P_e$) leading to an increase in δ

If $P_{e1} = P_{e0} + \Delta P$ then the accelerating power P_a decreases from ΔP (when $\delta = \delta_0$) to zero (when $\delta = \delta_1$).During the time taken by the load angle to increase from δ_0 to δ_1, the rotor absorbs KE. This KE equals to the shaded area A_1. At point 'b' the accelerating power is zero but the rotor has acquired a speed slightly greater than the synchronous speed and the angle δ continues to increase beyond δ_1. However as δ becomes greater than δ_1, P_a becomes negative causing the rotor to retard. The rotor swing continues till the load angle is δ_2 and the rotor attains a speed equal to synchronous speed.

Neglecting all losses (ex:-resistance, eddy current, damping etc..) the load angle δ_2 can be obtained from the condition that the KE gained by rotor during its swing from δ_0 to δ_1 must equal to KE returned as it swing from δ_1 to δ_2. This leads to conclusion that area A_1 must be equal to shaded area A_2. This is referred as equal area criterion.



Fig(7.10): Equal area criterion

The equal area criterion can be proved mathematically as follows:

From the swing equation

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M} = \frac{P_m - P_e}{M}$$

On multiplying the above equation by $2\left(\frac{d\delta}{dt}\right)$

$$2\left(\frac{d\delta}{dt}\right)\left(\frac{d^2\delta}{dt}\right)=2\left(\frac{d\delta}{dt}\right)\left(\frac{P_a}{M}\right)$$

$$2\left(\frac{d\delta}{dt}\right)\frac{d}{dt}\frac{d\delta}{dt}=2\left(\frac{d\delta}{dt}\right)\left(\frac{P_a}{M}\right)$$

$$2\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2=2\left(\frac{d\delta}{dt}\right)\left(\frac{P_a}{M}\right)$$

By taking integration on both sides

$$\left(\frac{d\delta}{dt}\right)^2=\frac{2}{M}\int_{\delta_0}^{\delta}P_a d\delta$$

$$\frac{d\delta}{dt}=\sqrt{\frac{2}{M}\int_{\delta_0}^{\delta}P_a d\delta}$$

For a stable system (i.e. the load angle will have minimum value when)

$$\frac{d\delta}{dt}=0 \quad \text{i.e.} \quad \int_{\delta_0}^{\delta}P_a d\delta=0$$

The physical meaning of the integration is the estimation of the area under the curve. Hence $\int_{\delta_0}^{\delta}P_a d\delta=0$ refers to zero area i.e the area under the curve P_a should be zero, which is possible only where P_a has both accelerating and decelerating power that is for a part of the graph $P_m>P_e$ and for the other part $P_e>P_m$ as shown in fig.(7.10)

For the generator action $P_m>P_e$ for positive area A_1 and $P_e>P_m$ for negative area A_2 for stable operation. Hence the name equal area criterion. The area A_1 represents the KE stored by the rotor during accelerations and the area A_2 represents the KE given by the rotor to the system and when it is all given up, the machine has returned to original speed.

7.12 APPLICATIONS OF EQUAL AREA CRITERION

7.12.1. Sudden change in Mechanical Input

The following fig.(7.11) shows the transient model of a single machine connected to infinite bus.

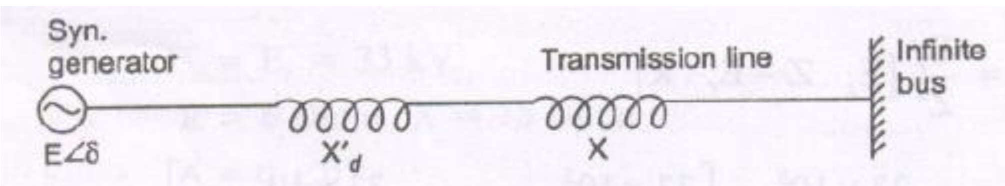


Fig.(7.11)

The electrical power transmitted is given by

$$P_e = \frac{|E||V|}{X}\sin\delta = P_{max}\sin\delta \qquad \text{---- (7.40)}$$

Under steady state operating condition

$$P_{e0} = P_{mo} = P_{max}\sin\delta_0 \qquad \text{---- (7.41)}$$

The power-angle curve of the generator is shown in the following fig.(7.12).In this the steady state point as described by eqn. (7.41) is the point 'a'.

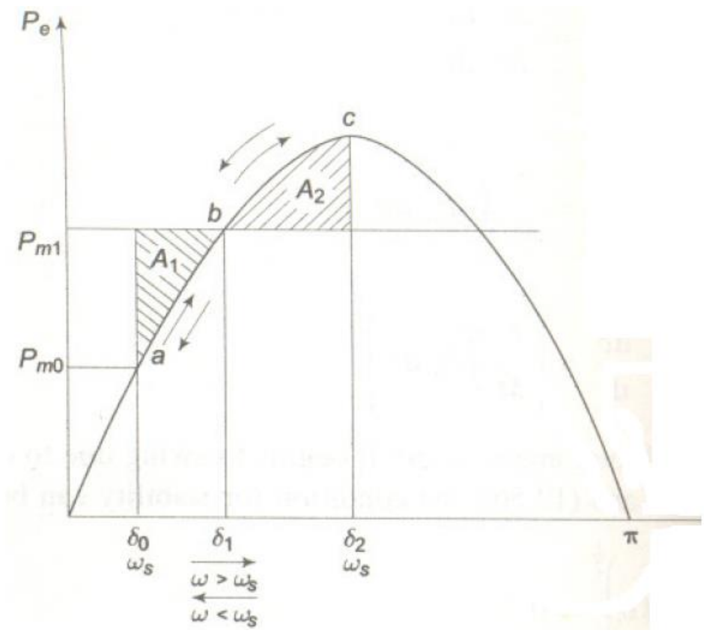


Fig.(7.12)

Let the mechanical input to the generator rotor be suddenly increased to P_{m1} (by opening the steam value). Since $P_{m1}>P_e$ the generator will have an accelerating power

$$P_a=P_{m1}-P_e \qquad \text{---- (7.42)}$$

Where $P_e = P_{max}\sin\delta$

Due to accelerating power the rotor speed increases and so the rotor angle also increases. This result in increased electrical power generation. Therefore the operating point will move upwards along the power-angle curve. At point 'b' again $P_{m1}=P_{e1}$, where P_{e1} is the electrical power output corresponding to torque angle δ_1. Now the rotor angle cannot stay at this point , because the inertia of the rotor will make the rotor to oscillate with respect to point b. Hence the torque angle will continue to increase till point 'c', when the operating point moves from b to c, $P_e>P_m$. There fore P_a (given by eqn.3) is negative and is called decelerating power. In this region (i.e. from b to c) the rotor speed decreases due to decelerating power. At point 'c' the speed of the rotor will be equal to synchronous speed.

Note: At point 'a' the speed is synchronous speed (ω_s). From 'a' to 'b' the speed increases and then from 'b' to 'c' the speed decreases. Once again at 'c' the speed is equal to ω_s. Thus the rotor oscillates between point 'a' and point 'c' before settling to point 'b'.

In fig.(7.12) A_1 is accelerating area and A_2 is decelerating area. The equal area criterion says that the system is stable, if

$$\int_{\delta_0}^{\delta_2} P_a d\delta = 0 \qquad \text{---- (7.43)}$$

To satisfy above eqn. (7.43), the accelerating area A_1 should be equal to decelerating area A_2. When the oscillations die -out the system will settle-down to a new state. In this new steady state $P_{m1} = P_{e1}$

$$\therefore P_{m1} = P_{e1} = P_{max} \sin\delta_1 \qquad \text{---- (7.44)}$$

The areas A_1 and A_2 can be evaluated as shown below

$$\left.\begin{aligned} A_1 &= \int_{\delta_0}^{\delta_1} (P_{m1} - P_e) d\delta \\ A_2 &= \int_{\delta_1}^{\delta 2} (P_e - P_{m1}) d\delta \end{aligned}\right\} \qquad \text{---- (7.45)}$$

From the above discussion we can say that there is upper limit for increase in mechanical power input P_m. As the mechanical power increased a limiting condition is finally reached at a point where the area A_1=A_2 as shown in fig.(7.13), the corresponding δ_1 can be δ_{1max} and δ_2 be δ_{2max}.

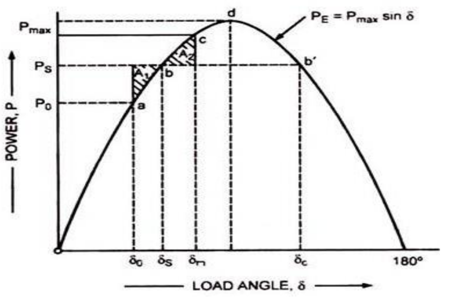


Fig.(7.13)

Here $\delta_{2max} = \pi - \delta_{1max}$ ---- (7.46)

From eqn.(7.40) $P_{e1max} = P_{m1max} = P_{max} \sin \delta_{1max}$

$$\therefore \delta_{1\max} = \sin^{-1}\left(\frac{P_{m1,\max}}{P_{\max}}\right) \qquad \text{---- (7.47)}$$

From eqns. (7.46) & (7.47)

$$\delta_{2\max} = \Pi - \sin^{-1}\left(\frac{P_{m1,\max}}{P_{\max}}\right)$$

From fig (7.13), we can say that any further increase in $P_{m1,max}$ will make the area $A_2 < A_1$. This means that the accelerating power is more than the decelerating power. Hence the system will have excess KE which causes s to increase beyond point 'c'. If the δ increases beyond point 'c' the decelerating power changes to accelerating power and so the system becomes unstable.

7.12.2. Critical clearing angle & Critical clearing time

The critical clearing angle, δ_{cc} is the maximum allowable change in the power angle δ, before clearing the fault, without loss of synchronism. The time corresponding to this angle is called critical clearing time, t_{cc}.

The critical clearing time, t_{cc} can be defined as the maximum time delay that can be allowed to clear the fault without loss of synchronism.

We see that for any given initial load there is a critical clearing angle. If the actual clearing angle is greater than the critical value the system is unstable, otherwise it is stable. So now we proceed to determine the value of the critical clearing angle for a given load.

Consider a single machine system as shown in fig.(7.14). Let the mechanical i/p be P_m and the machine is operating in steady-state with torque angle δ_0. The power-angle curve for this system is shown in fig.(7.15). The operating point is shown as point 'a'.

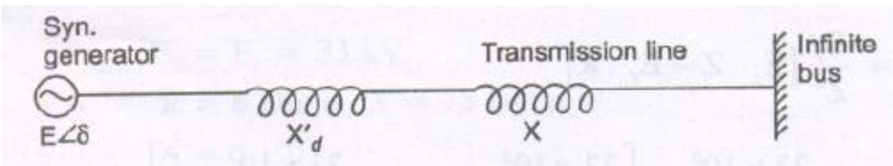


Fig.(7.14)

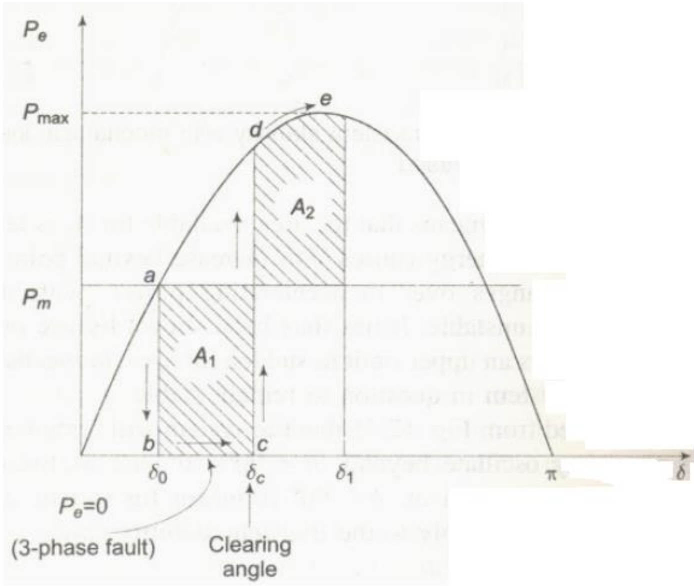


Fig.(7.15)

Let us assume a three phase fault occurs at point F in the system. Now P_e=0 and the operating point drops to 'b'. This means that the power transferred to infinite bus is zero and the entire power generated is flowing through the fault. Now the operating point moves along bc. Let the fault be transient in nature and so the fault be cleared by opening of the CB at point 'c', where $\delta = \delta_c$ and the corresponding time be t_c. Here t_c is called clearing time and δ_c is called clearing angle.

At time t_c corresponding to angle δ_c the faulted line is cleared by opening of the line circuit breaker. The system once again becomes healthy and the normal operation is restored. Now the operating point shifts to d . The rotor now decelerates and the operating point moves along de. For this transient state, if an angle δ_1, can be found such that A_2=A_1, then the system is found to be stable. The stable system may finally settle down to the steady operating point 'a' in an oscillatory manner due to damping in the system.

In the above discussion it is assumed that the fault is cleared at δ_c, but if the fault clearing is delayed then the angle δ_1 continuous to increase to an upper limit δ_{max}. This corresponds to a point where equal areas A_1 and A_2 can be found for a given P_m as shown in fig.(7.16). For this situation the fault would have been cleared at an angle δ_{cc} as shown in fig(7.16). This angle δ_{cc} is called critical clearing angle. The time corresponds to this angle is called critical clearing time t_{cc}.

If the fault is not cleared within critical clearing time, then δ_1 would increase to a value greater than δ_{max}. In such a situation the area A_2<A_1 and so the system would be unstable.

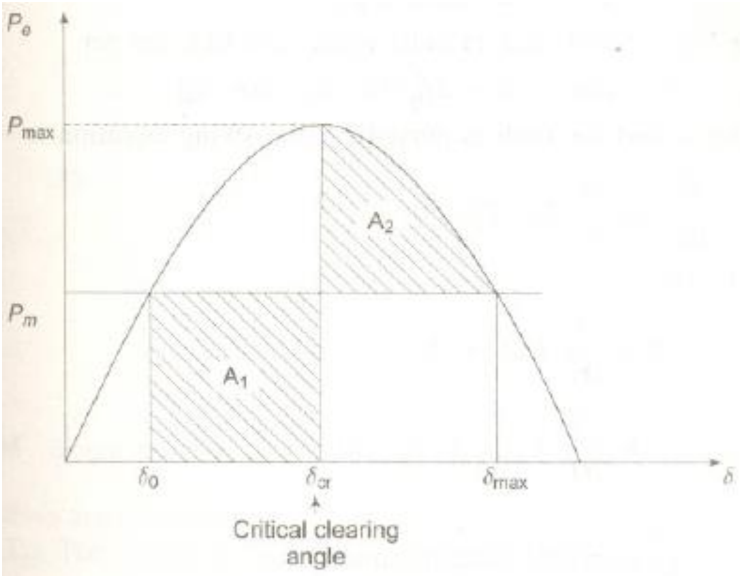


Fig.(7.16): Critical Clearing angle

Equations for δ_{cc} and t_{cc}:

For a 3-phase faults in simple systems, the equations for δ_{cc} and t_{cc} can be obtained as follows:

From fig (7.16), we can get

$$\delta_{max} = \pi - \delta_0 \qquad \text{---- (7.48)}$$

Under steady -state condition, for a given δ_0, $P_m = P_e$= constant

$$\therefore P_m = P_e = P_{max} \sin\delta_0 \qquad \text{---- (7.49)}$$

When a 3-phase fault occurs $P_e = 0$

$$P_a = P_m - P_e = P_m = \text{constant} \qquad \text{---- (7.50)}$$

The acceleration area A_1 is given by

$$A_1 = \int_{\delta_0}^{\delta_{cc}} P_m d\delta = P_m [\delta]_{\delta_0}^{\delta_{cc}} = P_m(\delta_{cc} - \delta_0) \qquad \text{---- (7.51)}$$

When the power feeding is reassumed after the fault

$P_e = P_{max} \sin\delta$

Now $P_a = P_{max} \sin\delta - P_m$ ---- (7.52)

The decelerating area A_2 is given by

$$A_2 = \int_{\delta_{cc}}^{\delta_{max}} (P_{max} \sin\delta - P_m) d\delta$$

$$= [-P_{max} \cos\delta - P_m \delta]_{\delta_{cc}}^{\delta_{max}}$$

$$= P_{max}(\cos\delta_{cc} - \cos\delta_{max}) - P_m(\delta_{max} - \delta_{cc}) \qquad \text{---- (7.53)}$$

For the stable system $A_1 = A_2$

$$P_m(\delta_{cc}-\delta_0)=P_{\max}(\cos\delta_{cc}-\cos\delta_{\max})-P_m(\delta_{\max}-\delta_{cc})$$

$$P_m\delta_{cc}-P_m\delta_0=P_{\max}\cos\delta_{cc}-P_{\max}\cos\delta_{\max}-P_m\delta_{\max}-P_m\delta_{cc}$$

$$P_{\max}\cos\delta_{cc}=P_{\max}\cos\delta_{\max}+P_m(\delta_{\max}-\delta_0)$$

$$\cos\delta_{cc}=\cos\delta_{\max}+\frac{P_m}{P_{\max}}(\delta_{\max}-\delta_0)$$

$$\delta_{cc}=\cos^{-1}\left[\cos\delta_{\max}+\frac{P_m}{P_{\max}}(\delta_{\max}-\delta_0)\right] \qquad \text{---- (7.54)}$$

Consider the swing equation of a single machine system

$$\frac{H}{\prod f}\frac{d^2\delta}{dt^2}=P_m-P_e \qquad \text{---- (7.55)}$$

During a 3-phase fault $P_e = 0$

$$\therefore \frac{H}{\prod f}\frac{d^2\delta}{dt^2}=P_m$$

$$\Rightarrow \frac{d^2\delta}{dt^2}=P_m\frac{\prod f}{H} \qquad \text{---- (7.56)}$$

Integrating the above equation (7.56) twice w.r.t 't'

$$\delta=\frac{P_m\prod f}{H}\frac{t^2}{2}+k$$

At $t = 0$, $\delta = \delta_0 \Rightarrow K = \delta_0$

$$\therefore \delta=\frac{P_m\prod f}{H}\frac{t^2}{2}+\delta_0 \qquad \text{---- (7.57)}$$

In equation (7.57) when $\delta=\delta_{cc}$, $t=t_{cc}$

$$\delta_{cc}=\frac{P_m\prod f}{H}\frac{t^2}{2}+k$$

$$t_{cc}=\sqrt{\frac{2H(\delta_{cc}-\delta_0)}{\prod fP_m}} \qquad \text{---- (7.58)}$$

7.12.3. Sudden Loss of one of the parallel line

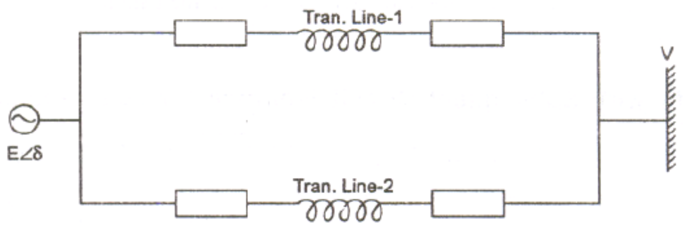


Fig.(7.17): Single machine connected to an infinite bus through two parallel line

Consider now a single machine tied to infinite bus through two parallel lines as shown in fig.(7.17).Equal area criterion can be used to study the transient ability of the system when one of lines is switched out. Two power angle diagrams are involved.

When both of the lines are operating the power transfer is given by

$$P_{e1} = \frac{EV}{X_{d_1} + \frac{X_1 X_2}{X_1 + X_2}} \sin\delta = P_{\max I} \sin\delta \qquad \text{---- (7.59)}$$

When one of the lines is switched out the transfer reactance increases and the power transfer is given by

$$P_{e2} = \frac{EV}{X_{d1} + X_1} \sin\delta = P_{\max II} \sin\delta \qquad \text{---- (7.60)}$$

$$\Rightarrow P_{\max I} > P_{\max II}$$

These two curves are as shown in fig. (7.18)

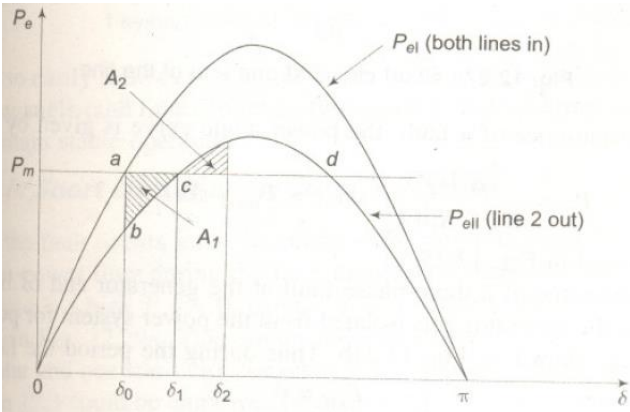


Fig.(7.18)

The i/p to the generator is P_m and , therefore the initial conditions are represented by point 'a' on curve I and the initial load angle is δ_0. When the line2 is switched out, the operating point shifts to point 'b' on curve II. At point 'b' the power o/p is less than the power i/p and the rotor is accelerated. An energy corresponding to area A_1 is put into rotor. At $\delta = \delta_1$ the i/p and o/p are equal but the angle continuously increased because the rotor has acquired a speed slightly greater than the synchronous speed. When $\delta > \delta_1$ deceleration of rotor starts. When $\delta = \delta_2$ area A_2 equals to area A_1 and the rotor starts swinging back. After a few oscillations the load angle will stabilize at δ_1. If the initial power transfer is increased (line P_m is shifted upwards in fig.(7.18) a limit is reached beyond which the decelerating area A_2 can't be equal to accelerating area A_1. The maximum value which can attain without loss of system stability is δ_m and equals to $(\pi-\delta_1)$ radians.

7.12.4. Fault and Subsequent circuit isolation

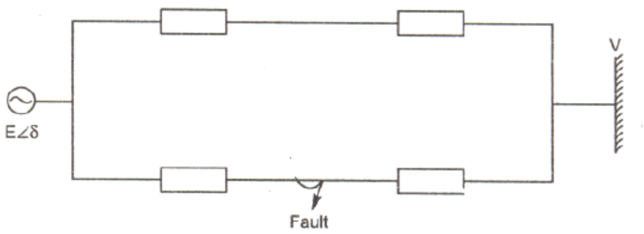


Fig.(7.19): SMIB system connected through two parallel lines

Consider the system shown in fig.(7.19). Equal area criterion can be used to study the stability of the system when a fault develops at any point F on line2 and is subsequently cleared by opening the CBS at both the ends of faulted line. In this case 3 power angle curves are involved, first for the pre fault system, second for the system during fault and third for the system after the fault line has been switched out(post fault condition). These curves are shown in fig.(7.20) and the input is P_m.

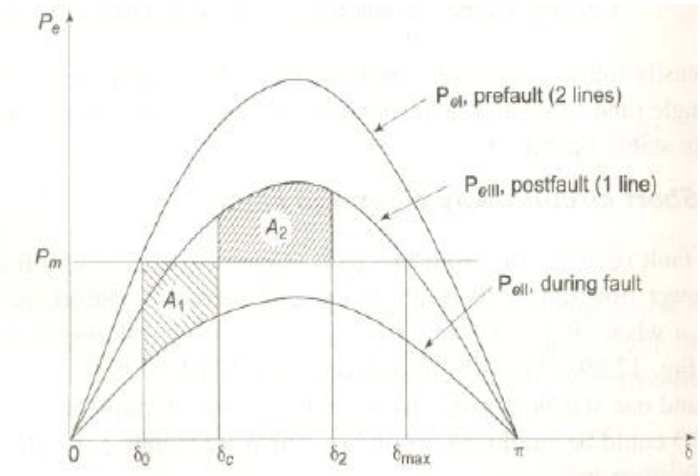


Fig.(7.20): Equal area criterion as applied to the stability study of fault and subsequent circuit isolation

The initial load angle δ_0 is determined by the intersection of the i/p line and the pre fault o/p curve (point a). When fault occurs the operating point shifts to 'b' on the o/p curve during fault. The accelerating power causes δ to increase. At angle δ_c when operation reaches point c, CBS opens and clears the fault. The operation shifts to point 'e' on the post fault curve. Now the o/p power is more than the i/p power and rotor starts decelerating. The max value of angle δ is δ_2 such that the area A_2 (=area defg) equals to area A_1(=area abcd).

A higher i/p P_m would cause point 'f' to move to right until at stability limit, point 'f' coincide with point 'h' as shown in fig.(7.21). A still higher value of P_m would lead to unstable condition. Another factor which would cause point 'f' to move to right is an increase in time of clearing the fault resulting in larger clearing angle δ_c.

For a given initial load there is a max value of clearing angle known as critical clearing angle δ_{cc} if the stability is to be maintained. If actual clearing δ_c is smaller the δ_{cc} system is stable, if larger, the system is unstable. When $\delta_c = \delta_{cc}$ the maximum angle up to which rotor swings is δ_{max} as shown in fig (7.21). An expression for δ_{cc} can be derived as under.

By applying equal area criterion to the above system

area A_1=area A_2.

$$\int_{\delta_0}^{\delta_{cc}} (P_m - P_{eII})d\delta = \int_{\delta_{cc}}^{\delta_{max}} (P_{cIII} - P_m)d\delta$$

$$\int_{\delta_0}^{\delta_{cc}} (P_m - P_{\max II} \sin\delta)d\delta = \int_{\delta_{cc}}^{\delta_{max}} (P_{\max III} \sin\delta - P_m)d\delta$$

$$\left[P_m\delta + P_{\max II} \cos\delta\right]_{\delta_0}^{\delta_{cc}} = -\left[P_{\max III} \cos\delta + P_m\delta\right]_{\delta_{cc}}^{\delta_{max}}$$

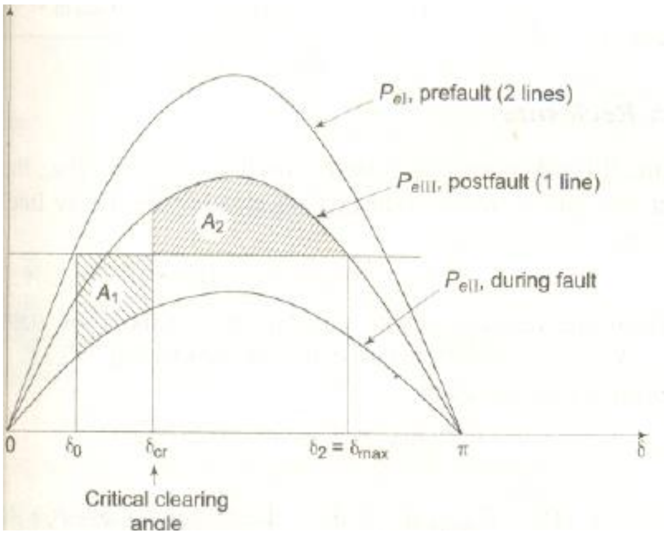


Fig.(7.21): Critical clearing angle

$$P_m(\delta_{cc} - \delta_o) + P_{\max II}(\cos\delta_{cc} - \cos\delta_0)$$
$$= -\left[P_{\max III}(\cos\delta_{\max} - \cos\delta_{cc}) + P_m(\delta_{\max} - \delta_{cc})\right]$$

$$P_m\delta_{cc} - P_m\delta_o + P_{\max II} \cos\delta_{cc} - P_{\max II} \cos\delta_0$$
$$= -\left[P_{\max III} \cos\delta_{\max} - P_{\max III} \cos\delta_{cc} + P_m\delta_{\max} - P_m\delta_{cc}\right]$$

$$\cos\delta_{cc}(P_{\max III} - P_{\max II}) = P_m(\delta_{\max} - \delta_0) + P_{\max III} \cos\delta_{\max} - P_{\max II} \cos\delta_{cc}$$

$$\cos\delta_{cc} = \frac{P_m(\delta_{\max} - \delta_0) + P_{\max III} \cos\delta_{\max} - P_{\max II} \cos\delta_{cc}}{P_{\max III} - P_{\max II}}$$

$$\delta_{cc} = \cos^{-1}\left[\frac{P_m(\delta_{\max} - \delta_0) + P_{\max III} \cos\delta_{\max} - P_{\max II} \cos\delta_{cc}}{P_{\max III} - P_{\max II}}\right] \qquad \text{---- (7.61)}$$

$$\text{Where } \delta_0 = \sin^{-1}\left(\frac{P_m}{P_{\max I}}\right) \qquad \delta_{\max} = \prod - \sin^{-1}\left(\frac{P_m}{P_{\max III}}\right)$$

7.12.5. Fault, Circuit isolation and reclosing

Most of the faults are of transient in nature. The transmission lines are provided with automatic quick reclosing circuit breakers. When a fault occurs the faulted line is de-energised to suppress the fault and reclosed after an interval to improve stability. Fig.(7.22) shows the application of equal area criterion for such a case. The i/p is P_m and initial load angle is δ_0.

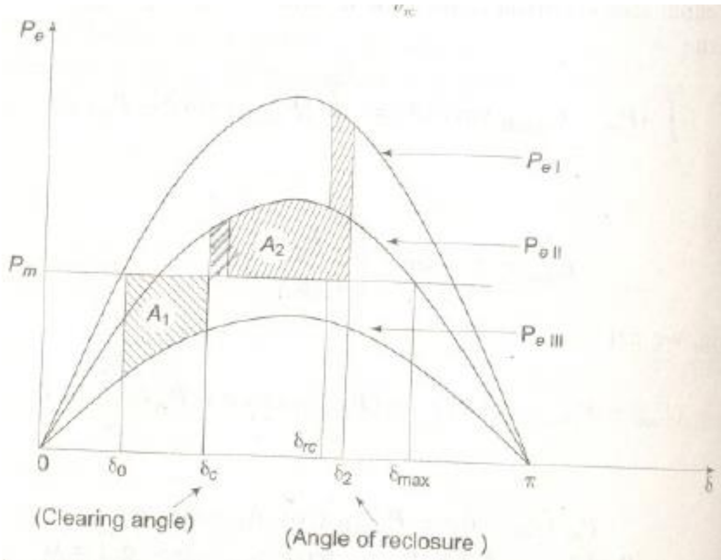


Fig.(7.22): Equal area criterion as applied to the stability study of fault, circuit isolation and reclosing.

When a fault occurs the operation shifts to the curve for faulted condition. When the load angle is δ_c, the faulted line is isolated and the operation shift to the post fault curve. When the load angle is δ_0 the circuit breaker reclose and operation shifts to pre-fault curve. For stable operation the accelerating area A_1 should be equal to decelerating area A_2. The maximum angle to which rotor swings is δ_2 and is less than δ_m (i.e the maximum permissible rotor swing if stability is to be maintained).

7.13 SOLUTION OF SWING EQUATION BY POINT BY POINT METHOD

The swing equation, a differential equation governing the motion of each machine of a system is

$$M\,\frac{d^2\delta}{dt^2} = P_a \qquad \text{---- (7.62)}$$

The solution of the above equation gives load angle' δ ' as a function of time *'t'*. A graph of the solution of the swing equation is called 'swing curve'. Inspection of the swing curves of all the machines of a power system will show whether the machine will remain stable or unstable after a disturbance.

The formal solution of such system gives a set of non-linear differential equations. The point by point (or step by step) method is the most feasible and widely used way of solving the swing equations. In this method one or more variables are assumed either to be constants or to vary according to assumed laws throughout a short interval of time' *M'*, so that as a result of assumptions made the swing equation can be solved for the changes in the other variables during the same time interval.

The main assumption for solving the swing equation by point by point method is "the accelerating power is constant during time interval" .

Integrating twice, w.r.t. time't' of equation (7.62) and can be modified as

After 1st integration, $\frac{d\delta}{dt} = \omega = \omega_0 + \frac{P_a}{M} t$ ---- (7.63)

After 2 nd integration, $\delta = \delta_0 + \omega_0 t + \frac{P_a}{2M} t^2$ ---- (7.64)

These two equations gives the ' ω', the excess of speed of the machine over normal speed and 'δ', the angular displacement of the machine with respect to reference axis rotating at normal speed. Here δ_0 and ω_0 values are angular displacement and angular velocity at the beginning of the interval.

Dividing the total time 't' into 'n' equal intervals. Let subscript 'n' denote quantities at the end of interval, from equations (7.63) and (7.64), we get

$$\omega_n = \omega_{n-1} + \frac{\Delta t}{M} P_{a_{n-1}} \quad \text{---- (7.65)}$$

$$\delta_n = \delta_{n-1} + \Delta t . \omega_{n-1} + \frac{(\Delta t)^2}{2M} Pa_{n-1} \quad \text{---- (7.66)}$$

The increments of speed and angular displacement during the nth interval

$$\Delta\omega_n = \omega_n - \omega_{n-1} = \frac{\Delta t}{M} Pa_{n-1} \quad \text{---- (7.67)}$$

$$\Delta\delta = \delta_n - \delta_{n-1} = \Delta t . \omega_{n-1} + \frac{(\Delta t)^2}{2M} Pa_{n-1} \quad \text{---- (7.68)}$$

The equations (7.65) and (7.66) or (7.67) and (7.68) are suitable for point by point calculation. However, if one is interested only in the angular position but not in the speed, ω_{n-1} can be eliminated in equations (7.66) and (7.68) and write an equation like, equation (7.66) but for previous interval

$$i.e.\delta_{n-1} = \delta_{n-2} + \Delta t.\omega_{n-2} + \frac{(\Delta t)^2}{2M} Pa_{n-2} \qquad \text{---- (7.69)}$$

From the equations (7.66) and (7.69)

$$\delta_n - \delta_{n-1} = (\delta_{n-1} - \delta_{n-2}) + \Delta t.(\omega_{n-1} - \omega_{n-2}) + \frac{(\Delta t)^2}{2M}(Pa_{n-1} - Pa_{n-2}) \qquad \text{---- (7.70)}$$

But we know that

$$\delta_n - \delta_{n-1} = \Delta\delta_n$$
$$\delta_{n-1} - \delta_{n-2} = \Delta\delta_{n-1}$$
$$\omega_{n-1} - \omega_{n-2} = \Delta\omega_{n-1}$$
$$\Delta\omega_{n-1} = \frac{\Delta t}{M} Pa_{n-2}$$

Substituting these values in equation (7.70)

$$\Delta\delta_n = \Delta\delta_{n-1} + \Delta t.\frac{\Delta t}{M}.Pa_{n-2} + \frac{(\Delta t)^2}{2M}(Pa_{n-1} - Pa_{n-2})$$

$$\therefore \Delta\delta_n = \Delta\delta_{n-1} + \frac{(\Delta t)^2}{2M}(Pa_{n-1} + Pa_{n-2}) \qquad \text{---- (7.71)}$$

This equation, which gives the increment in angle during any interval in terms of the increment for the previous, may be used for point by point calculations in place of equations (7.67) and (7.69). The Lost term of the 2nd differential of 'δ' which may symbolized by '$\Delta^2\delta$' The time interval 'Δt' should be short enough to give required accuracy but not so short so as to un delay increase the number of point to be computed on a given swing curve.

7.14 METHOD OF IMPROVING STABILITY

1. By increasing inertia constant(M)

From the swing equation $\frac{d^2\delta}{dt^2} = \frac{P_a}{M}$, it is obvious that for given accelerating power the acceleration of the rotor is inversely proportional to the moment of inertia of the machine i.e. the higher the moment of inertia the slower will be the change in the rotor angle of the machine and thus will allow a longer time for the operation of the breaker to allow a longer time for the operation of the breaker to isolate the fault before the machine passes through the critical clearing angle. Hence transient stability can be improved either by using machines of higher inertia or by connecting the synchronous motors to heavy fly wheels.

However this cannot be employed in practice because of economic reasons. Also increasing 'M' will have an undesirable effect of slowing down the response of the speed governor loop.

2. Increasing system voltage

$$P_e = P_{max} \sin\delta = \frac{EV}{X}\sin\delta$$

Transient stability is improved by increasing system voltage. Increase in system voltage means the higher value of max. power, P_{max} that can be transferred over the lines. Since shaft power $P_e = P_{max} \sin\delta$,therefore for a given shaft power initial load angle δ_0 reduces with the increase in P_{max} and thereby increasing difference between the critical clearing angle and initial load angle. Thus machine is allowed to rotate through large angle before it reaches the critical clearing angle which results in greater critical clearing time and the probability of maintaining stability.

3. Reduction of transfer reactance

The steady state power limit is given by

$$P_{\max} = \frac{EV}{X}$$

It can be seen from this expression that P_{max} can be increased by increasing either or both V and E and reducing the transfer reactance . The following methods are available for reducing the transfer reactance.

a. Use of double circuit lines

The impedance of a double circuit line is less then that of a single circuit line. A double circuit line doubles the transmission capability. An additional advantage is that the continuity of supply is maintained over one line with reduced capacity when the other line is out of service for maintenance or repair. But the provision of additional line can hardly be justified by stability considerations alone.

b. Use of Bundle conductors

The use of bundle conductors is another method of reducing reactance. Bundle conductor line are generally used in EHV system for transmitting bulk power over long distances. The main purpose of employing bundle conductors in such line is to reduce corona loss and radio interference.

c. Series compensation of the lines

One method of reducing the reactance of the lines is by employing capacitors in the lines and such lines are known as series compensated lines . Series capacitors are employed in EHV lines to increase the power transfer and are most economical for transmission distance more than 350km.

4. Fast switching

Rapid isolation of faults is the principle way of improving transient stability. The fault should be cleared as fast as possible. It should be noted that the time required for fault removed is the sum of relay response time plus the CB operating time.

Therefore, high speed relaying and circuit breaking are commonly used to improve stability during fault conditions. It has now become possible to isolated the fault in less then two cycles(i.e.0.04sec for 50 Hz system)

RECENT METHODS

5. Turbine fast valving (or) By-pass valving

When the fault occurred , the generator output is reduced resulting in a high accelerating power. If the mechanical input power to the turbine should be momentarily reduced, the acceleration could be reduced . Fast valving is a means of reducing the mechanical input power to the turbine during the fault .Certain steam valves are rapidly closed (in 0.1 to 0.2 sec) and immediately reopened. This procedure increases the critical clearing time.

6. Single –pole switching

Majority of the line faults are LG faults. In single pole switching (also called independent pole operation), the three phase of the CB are closed or opened independently of each other. In the event of an LG fault, the circuit breaker pole corresponding to the fault line opened and the remaining two healthy phases continue to transfer power. Since most of the faults are transient in nature, this phase can be reclosed after it has been open for a predetermined time. The system should not be operated for long periods with one phase open. Therefore provision should be made to trip the whole line if one phase remaining open for a predetermined time.

7. Load shedding

If there is in sufficient generation to maintain system frequency, some of the generators are disconnected during or immediately after a fault. Thus, the stability of the remaining generators is improved. The unit to be disconnected is provided with a large steam by pass system. When the system recovers from the shock of the fault, the disconnected unit is resynchronized and reloaded. Extra cost of a large steam by pass system is the limitation of this method. Disconnection of some of consumers, that is, load shedding (removed of load), is also helpful in improving transient stability.

8. HVDC links

Increased use of HVDC links employing thyristors would alleviate stability problem. A dc link is asynchronous i.e. the two ac system at either end do not have to be controlled in phase or even be at exactly the same frequency as they do for an ac link, and the power transmitted can be readily controlled. There is no risk of a fault in one system causing loss of stability in the other system.

9. Breaking resistors

For improving stability where clearing is delayed or a large load is suddenly lost, a resistive load called a braking resistor is connected at or neat the generator Bus. This load compensates for at least some of the reduction of load on the generators and so reduces the acceleration. During a fault the resistors are applied to the terminals of the generators through circuit breakers by means of an elaborate control scheme. The control scheme determines the amount of resistance to be applied and its duration. The braking resistors remain on for a matter of cycles both during fault clearing and after system voltage is restored.

10. Short circuit current limiters

These are generally used to limit the short circuit duty of distribution lines. These may also be used in long transmission lines to modify favourably the transfer impedance during fault conditions so that the voltage profile of the system is somewhat improved, thereby raising the system load level during the fault.

11. Full load rejection technique

Fast valving combined with high speed clearing time will sufficient to maintain stability in most cases. However, there are still situation where stability is difficult to maintain. To remedy this situation, a full load rejection scheme could be utilised after the unit is separated from the system. To do this , the unit has to be equipped with a large steam by pass system. To do this, the unit has to be equipped with a large steam by pass system. After system has recovered from the shock caused by the fault, the unit could be resynchronized and reloaded. The main disadvantage is the extra cost of the large by pass system.

7.15 HIGH SPEED CIRCUIT BREAKER- RECLOSING OR AUTO RECLOSING CIRCUIT BREAKERS

High speed circuit breaker reclosing is a method of automatically operating CBS according to a predetermined sequence of open and close operations. When a fault occurs on a transmission line, the CB at each end open, to isolate the line, remain open for specified time(delay time) and then reclose. If the fault has cleared, then the CBS remain closed and the transmission system returns to its pre-fault condition. If the fault still exist the CBS open and lock-out.

The effect of high speed CB reclosing can be explained as follows:

During a fault, the generator rotor angle can change relative to rest of the system until a critical angle δ_c is reached. If the rotor angle exceeds the critical angle the generator is no longer synchronized with the system i.e. it is unstable. When a fault occurs on the transmission system the output power of any generator near the fault is reduced, the input power remains constant and the difference between input and output power accelerates the generator . If the fault is not cleared, the generator will continue to accelerate until it pulls out of synchronism and trips off due to protective relay action. If the CBS open, clear the fault and reclose successfully, load is reapplied to the generator and the energy which was stored in accelerating the generator is released into the system. If this is done before the critical angle is passed, the generator decelerate and re-establishing its stable pre-fault condition. For reclosing to be successful the CB must remain open long enough for the integrity of the transmission line insulation level to be re-established, and it must reclose before the generator can accelerate beyond the critical angle. Detailed studies are required to stability limits w.r.t. fault clearing time and delay time. If the stability limit are not accurately determined, instability may result/

Automatic reclosing is used because approximately 80% of transmission line faults are transient in nature i.e. if the line is de-energized for a short time. The fault is will de-ionized and the integrity of the insulation system will be re-established.

ADDITIONAL SOLVED PROBLEMS

Problem-1: A Generator supplies power to an infinite bus via a transmission line as shown in figure. The machine delivers 1.0pu power and both terminal voltage of the generator and infinite bus are 1.0p.u .All the reactance's are on common base. Determine the power angle equation.

Solution: The transfer reactance between V_t and V is

$$X = j0.1+j0.4 = j0.5$$

$$P_e = \frac{|V_t||V|}{X}\sin\theta = \frac{1\times 1}{0.5}\sin\theta = 1 \Rightarrow \theta = 30^0$$

Terminal voltage,

$$V_t = 1\angle 30 = 0.866 + j0.5$$

Current supplied by the generator to the infinite bus is

$$I = \frac{0.866 + j0.5 - 1}{j0.5} = \frac{-0.134 + j0.5}{j0.5} = 1 + j0.268 = 1.0353\angle 15^0$$

Transient internal voltage in the generator

$$E = 0.866 + j0.5 + j0.2(1 + j0.268) = 0.812 + j0.7 = 1.07\angle 40.76$$

Total X=j0.2+j0.1+j0.4=j0.7

The power angle characteristic is

$$P_e = \frac{|E||V|}{X}\sin\delta = \frac{1.07\times 1}{0.7}\sin\delta = 1.528\sin\delta$$